THE BLOTTING PAPER THEORY OF TRANSPORT BY LARGE EDDIES

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If I want to soak up ink with blotting paper, I place the blotter on the surface of the paper, wait while the ink diffuses up into the blotter, then take the blotter away again. Both the small-scale process, the diffusion into the paper, and the large-scale process, the movement of the blotter, are necessary if the blotting action is to be successful. If the blotting paper is impermeable (zero diffusivity) then the excess ink will go nowhere. If the blotter is left stationary on the paper then ink will only go a little way and the large-scale motion will not have contributed to the result. Blotting is a two-scale process: fine-scale diffusion and coarse-scale bulk movements are both necessary for successful removal of ink.

So it is with transport by large eddies, though this is multi-scale. It is not enough that large masses of air move about in the atmosphere, they must also take up scalar substance if they are to carry it anywhere. The taking-up part is by the smaller-scale motions and the carrying-away is by the larger-scale motions. Let's classify 'eddies' into three groups: those whose height scale is much less than height above ground, those whose height scale is much greater than height above ground, and a group in the middle whose height is comparable to height above ground. Let's just think about the extremes to begin with.

Let us watch a large eddy doing its work, and take the view of a Mr Lagrange, who reclines on a magic carpet and floats along close above the ground with a large air mass which is, for the while, moving horizontally. Mr Lagrange's ride is a little bumpy as the small eddies move about, carrying scalars from the ground up past him. He notices that these small motions move air parcels about over quite small distances, so their effect is to 'diffuse' the scalar down the local concentration gradient which has formed about him. He notices the scalar concentration increasing with time as his air mass picks up more and more of it from below. By now Mr Lagrange is starting to rise gently. He passes above a Mr Euler, who is on the ground. Mr Euler is rushing distractedly about, attending a complicated set of micrometeorological instruments on a mast. Mr Euler pauses for a moment as Mr Lagrange floats overhead and also notices that the scalar concentration is increasing. He interprets it spatially as a streamwise temperature gradient in one of Mr Taylor's frozen eddies, before he dashes off to find out why the diesel generator is running roughly. Soon after, Mr Lagrange finds his air mass rammed from behind by a faster-moving air mass, his having dragged along the ground and slowed down. His air mass is pushed upwards and Mr Lagrange is last seen disappearing into the sky in the midst of his scalar-rich air.

Meanwhile Mr Euler labours on. Many months later, after dealing with an internal review of his laboratory and consequent restructuring, Mr Euler finally looks at his data. He plots out his heat flux cospectra and finds flux at high and low frequencies. He decides to try and parameterize these. The high-frequency flux presents no problem: he models it as a diffusion process, multiplying the local concentration gradient by an eddy diffusivity. The eddy diffusivity he parameterizes in terms of surface-layer parameters z and u, as has been done by many...
before him, though Mr Euler has to replace the von Karman constant with his own value because he deals with only the small-eddy part of the spectrum. The slow fluctuations present more of a problem because Mr Euler knows the diffusion analogy doesn't work for big eddies. He wonders what he should do?

To get some ideas he goes to his turbulence textbook, written by Mr Hinze, and starts at the beginning with equation (1.1). To his surprise he discovers that, at the most basic level, the flux of and transferable property is described solely as advection. He recalls that, during the latter half of the ramps, he noticed persistent updrafts and vertical scalar gradients at the same time. Advection perhaps? Not in the mean, he knows, because the mean w is zero, but a fluctuating kind of advection perhaps? He copies out (1.1) from Mr Hinze's book and does a Reynolds expansion on the velocity and 'transportable property' variables. He looks at the book again to check progress, but finds that Mr Hinze has got rid of the fluctuating parts of the advection terms without ever writing them down, having transformed them into divergences of the Reynolds flux by fiddling about with the continuity equation. Disappointed, Mr Euler goes to the library to find a better book, but finds they all do the same thing!

Mr Euler is now on his own. He decides to make it easier for himself so he does the sums only for a uniform surface layer. He averages his expanded equation and sets all the time-varying and mean-advection terms to zero, to find that only the three components of fluctuating advection remain, which therefore have a zero sum. He looks at these terms carefully but can find no reason why the individual terms are zero. It seems that whatever scalar is picked up by the horizontal fluctuating advection is dissipated by the balancing vertical term. After some reflection he wonders if the horizontal terms are related to the ramps he noticed in the field, and so perhaps they represent the accumulation of scalar in the air mass. If so, there must have been fluctuating amounts of scalar in storage, he reasons, but the term for this has disappeared in the averaging process. He will have to ask Mr Lagrange about that.

Mr Euler decides to think about blotting paper, to help him get his ideas straight. In blotting paper the ink is carried up past an observation point by diffusion. The gradient in ink concentration is a characteristic of that diffusion. The large vertical motions, on the other hand, are analogous to movements of the blotting paper as it is lifted up. As the blotter is lifted, he reasons, the ink from below the observation point is carried upwards and can be calculated as advection. Some of the ink absorbed from the surface into the blotter had already diffused upwards past his observation level and was already accounted as the high-frequency part of his flux. What he has to add is the ink from below passing up with the bulk motion of the blotter. With this in mind he looks at his field data, and at the spectra from the various levels on his mast. He is not surprised to find that, as height increases, progressively less and less flux is carried by small-eddies and progressively more and more by the large eddy motions. That is, he notices flux in the low-frequency part of his flux spectrum growing with height while the flux in the high-frequency part reduces by the same amount. The spectrum is shifting to lower frequencies.

Carefully pondering these matters, Mr Euler observes that vertical velocity of the eddies scale on the large-scale motions. He chooses the symbol v, for this. He also reasons that the scalar fluctuations depend on the small-scale diffusion process and so must be related to some measure of the local scalar profile. He knows that Misters Monin and Obukhov had used the scalar flux divided by u∗ as their scale, but Mr Euler is not happy with this because he doesn't know how much of the total flux is carried by the small eddies characterized by u∗. The only local quantity that seems to be clearly related to the profile is the mean scalar gradient, ∂<s>/∂z. For dimensional reasons he adds a z to his list of parameters. Doubtfully, he writes his result as v∗∂<s>/∂z. At least this has the dimensions of a scalar flux and expresses an interaction of large and local scales.

Mr Euler looks at his result cautiously and is surprised to find that, with this parameterization, the flux carried by both large and small eddies is down the local scalar gradient, just as in classical diffusion. The novelty is that a small-eddy velocity scale parameterizes the diffusivity of the small eddies while a large-eddy velocity scale parameterizes the 'diffusivity' of the larger ones.
To test his deduction, Mr Euler looks at cases where the velocity scales for large and small eddies are different. He looks at cases where some kind of external disturbance puts extra energy into the large eddies, and he notices more of the flux is carried at low frequencies in those cases. He also notices the scalar gradients are smaller relative to the total flux in the 'disturbed' cases, because more of the flux is carried by the larger motions and less by the small-scale diffusion. That is, he notices that disturbance increases 'eddy diffusivities'. Mr Euler is now better satisfied with his work, which seems to explain many features of the scalar flux spectrum in disturbed and undisturbed conditions.

After all this mental effort, Mr Euler is quite tired and decides not to worry about intermediate-scale eddies right away. He goes off to relax in a nice hot bath. Unfortunately, he finds Mr Archimedes there already, floating in it with an amount of water equal to his weight spilled onto the floor. C'est la vie.

PS: I wrote this story while trying to understand the low frequency part of turbulence spectra. I had been thinking about it for a while and decided to summarize where I had got to. His couldn't be too formal because my ideas were still very loose. Mr Euler and Mr Lagrange emerged as shorthand for Eulerian and Lagrangian frames of reference, and soon I began to enjoy myself. I was rather pleased with the result so I saved it at the end of the session. Now (a year later) I recognize mistakes as well as insights in the physics. Consider it as an entertainment.